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SIMULATION AND ANALYTIC  
MODELS OF MEMORY

by

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## SIMULATION AND ANALYTIC MODELS OF MEMORY

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A scientific model summarizes our knowledge about particular phenomena and permits us to make predictions about certain situations. As understanding of a subject progresses we demand increasingly accurate summaries and increasingly detailed predictions. Galileo could be interested in the question of whether two bodies of unequal mass fall at the same rate; today the effect of the pressure of sunlight must be calculated. The literature of the 1930s and 1940s is full of experiments testing predictions of order relationships -- "Would the group with treatment x learn more or less rapidly than the control group?" Today, mathematical theories are used to predict the precise distributions of responses to a given situation. Predictions are now made on a ratio scale.

The increased precision characteristic of modern learning theory has not yet been matched by theories of human memory. Many of our experiments are still concerned with order relationships. This is not surprising. Human memory phenomena are much more complex than, say, binary choices. Precision will come more slowly to the more complex field, but it will come. We have had examples of analytic models of memory, and we shall have more in the future. But because of the complexity of memory, construction and evaluation of models will be difficult. The problems are basic. They have their genesis in the nature both of models and of human memory.

Strictly speaking, a model is no more than the abstract mathematical statement used to specify a mapping from some stimulus parameters to some response parameters. Experiments test the accuracy of the mapping. They do not test the validity of the reasons for proposing the mapping. The mathematical statements, however, are usually abstract descriptions of some process that has a psychological interpretation. Restle (1959) has pointed out that most stimulus sampling models of learning are of this nature. He further argues that having such an underlying process in mind is useful, not because it increases the precision of a theory but because it serves as a useful heuristic to suggest experiments and future elaborations. In studying human memory, several processes must be envisaged. We know that they are necessary because people can store information. So can a library, a magnetic tape, a dictionary, or a notebook. Because we live in the age of the "information explosion" (among other explosions), a good deal of attention has been paid to the processes which must be introduced in any information storage system. A minimum set of information handling processes is specified in Figure 1.

Any information presented to a storage system must be coded into a form that the system can handle. What this form is will be determined more by the physical characteristics of the system than by the information transmitted. We do not store light pulses; we store nerve pulses and perhaps RNA molecules. The internally stored, coded form of the stimulus must be composed

of things that the storage system can handle as a single unit. Miller (1956) has referred to the "chunk" of information, the unit which the human can manipulate, as distinct from the bit, a measure of mathematically defined information in a signal. I shall assume that every stimulus is somehow transformed into a set of chunks, without specifying what these chunks are. Any storage system must have a place to put its chunks. In libraries these are shelves. Presumably humans have a place for memory somewhere in their brains. Clinical evidence on direct stimulation of the cortex (Penfield and Roberts, 1959) suggests that someday we will know where it is. For the present, all we have to do is assume that the storage unit does exist. An information storage system also must have a filing system. This is a record of the contents of the storage units. It is needed so that an efficient search of the storage units may be made. The optimal organization of such records is an unsolved problem. One alternative is to have a centrally located file, as in a library. Another alternative is to organize the files in a manner similar to road signs, on a "From here go west to get to X" basis. This seems to be a more reasonable model of a psychological process.

When a question is presented to an information storage system, it too, must be coded. The purpose of this coding is not storage. The chunks contained in a question are used to set up requirements for a route through the information storage system. This route is used to assemble those stored chunks which answer the question at hand. The assembled chunks must then be recoded

back into an observable response. The actual route used in assembling chunks will be established by consulting the files for an itinerary that appears to be satisfactory. Unless there is a perfect correspondence between the files and the actual state of the storage unit, the planned itinerary may not find the correct chunks. This situation is somewhat familiar to the users of libraries. It is equally familiar to anyone who was sure of the answer to a question but was wrong. An intermediate position is also possible. The planned route may assemble some chunks that can be positively identified as relevant and others whose veracity is known to be under question.

This managerial description of information retrieval does not sound particularly psychological, but there is a place for the phenomena of human memory within this descriptive scheme. Particular discrepancies between storage units and files will lead to characteristic errors in information recovery. The spaces into which stimulus chunks may be placed will be a function of the amount of space being utilized at the time. The chance of a particular chunk of information being in the storage unit the filing system thinks it is in will be a function of the amount of change the storage unit has been subjected to since the file was last checked. Response bias and proactive and retroactive interference effects are not limited to living organisms, they are inherent in information storage.

Obviously, there are quite a few steps between stimulus and response. Since each of them represents a possible slip, it is

hardly surprising that we do not have a model of memory in its entirety. Several models of sub-steps have been offered. By and large, these have concentrated on the information storage and retrieval processes. The hypothetical processes generate workable (and even elegant) mathematical abstractions, from which theorems about behavior of the coded stimulus and response can be derived. Unfortunately, the model itself cannot be tested unless either a one to one relationship between external stimulus and internal code can be assumed, or unless a very simple model of coding is postulated for application only in limited situations. If models for every process were to be developed independently, combining them would be difficult. The combination might no longer lead to a comprehensible set of mathematical equations. In this case, it may be advisable to skip the mathematical abstraction stage. A general model of memory can be constructed physically by appropriate programming of a general purpose computer. Experiments can be simulated and the results of the simulation compared to the results of actual experiments using human subjects. This process has been advocated for the study of problem solving (Newell, Shaw and Simon, 1958). It is potentially useful in studying memory.

Programming does not solve the problems raised in evaluating mathematical models, it merely pushes them back. No foreseeable simulation model, programmed or otherwise, will contain a representation of every step involved in memory. Analytic and simulation models are not different in kind; they represent points on a continuum of simplicity-complexity, both as regards to their own

structure and to the situations in which they may be applied. The following two samples have been chosen to illustrate this point.

#### A Mathematical Model of Recognition

To illustrate the mathematical approach, I have reworded a model proposed by Shepard (1961) for the retention of information from a sequence of presentations of stimuli. The adaptation I propose regroups his assumptions to agree with the classification of memory processes listed in Figure 1.<sup>3</sup>

#### Coding Assumptions

(a) Every stimulus is represented internally by the set of stimulus elements (chunks) located at a particular reference point. The individual chunks are undifferentiated.

(b) When a stimulus is presented, n chunks are activated and stored at that stimulus' reference point.

These assumptions can be interpreted as saying that each stimulus is represented by the chunks stored in locations marked with the name of that stimulus (e.g. a library shelf). The locations remain constant, but their contents do not. The coding assumptions reduce possible tests of the model to those situations in which a single stimulus can reasonably be represented by a set of uniform elements.

#### Storage Rules

(a) On every stimulus presentation there is a probability, u, that a given chunk will remain active (i.e. be in storage at all).

(b) At each stimulus presentation there is a probability,  $v_{ik}$ , that a chunk will "migrate" from the reference point representing stimulus  $i$  to that representing stimulus  $k$ . The values of  $v_{ik}$  satisfy the following further constraints:

(b.1)  $v_{ii} \geq v_{ik}$ , the most likely event is that a chunk stays where it is.

(b.2)  $v_{ik} = v_{ki}$ , migration is symmetric.

(b.3) There are only two possible values of  $v_{ik}$ , one for migration between similar stimuli and one for migration between dissimilar stimuli. Psychologically, this means that stimuli fall into clusters of similar items.

#### Retrieval Rules

The subject will be shown a sequence of stimuli, (the inspection sequence), and then will be presented with a small array of stimuli, (the ). He will be asked which of the items in the array were in the inspection sequence. The rules by which this question is to be answered are:

(a) The coded form of the question is simply the name of the reference point for that stimulus to which the subject must respond.

(b) All the active chunks in the corresponding shelf are assembled when a stimulus is presented for recognition.

(c) The unbiased probability,  $P^*$ , that a subject will call a particular stimulus "old" is proportional to the number of active chunks on the corresponding shelf.

(c.1) The stimulus whose shelf contains the most active chunks will be called "old" with probability one.

(c.2) The actual probability  $\underline{P}$  of responding to a stimulus as "old" is a power function of the unbiased probability,  $\underline{P} = (\underline{P}^*)^B$   $B > 0$ .

These assumptions limit the type of stimuli that can be used to uniform stimuli which fall into natural clusters of similar items. The range of questions that can be asked is similarly limited. These are the boundary conditions within which we can test the implications of the assumed storage process. When the boundary conditions are satisfied, it is possible to develop specific statements of the probability of the subject's responding to a stimulus as an "old" stimulus, both when it was not in the inspection sequence and when the stimulus was in the sequence with  $d$  items intervening between presentation and test. The appropriate equations and terminology are shown in Table 1.

Shepard's model is not only precise, it is accurate. Shepard and Teghtsoonian (1961) showed that the model will predict a subject's performance when he is shown a sequence of three digit numbers and then asked to identify those numbers in an array of numbers which were also in the sequence. From this experiment we are entitled to draw one conclusion from a strict interpretation of the data; Shepard's model is an accurate representation of some of the information processing accomplished by humans when the boundary conditions are satisfied. We are also likely to draw the extra-logical conclusion that something resembling the underlying storage process operates in human memory. However, just how the model could be extended to predict in more complex situations is

not clear. If more than two levels of stimulus similarity are involved, the number of parameters to be estimated from the data increases rapidly. The model itself contains no process for recovering information about the order in which stimuli were presented; yet humans certainly retain such information. Many interesting human memory tasks involve recognition of old stimuli and other things as well. The model may be extended to a variety of more complex memory experiments. At some point, it will become virtually impossible to develop workable mathematical expressions to describe how a person should behave.

#### Simulation of the Keeping Track Task

The second example is of a model for a more complex situation, the keeping track task. As in Shepard's situation, the subject receives information by observing a sequence of stimuli. The stimuli represent the changing states of variables in the subject's environment. His job is to remember the current state of each variable. A practical example of a keeping track task is the job of an air traffic controller. He receives a series of messages; "Flight 714 is low on fuel," "Flight 622 is heading south," "Flight 509 is descending to 10,000 feet," etc. Aperiodically he must respond to a question, "What is the direction of Flight 622?" The experimental situation mirrors this. The subject must keep track of meaningful attributes of arbitrary objects, such as "Animal of A."

Keeping track situations can be described by stating certain parameters, the number of variables in the environment, the way in

which they are divided into attributes of objects (e.g. the altitude and direction of Flights x, y, z), the number of possible states per variable, the rate at which variables change their states, the frequency of questions, and the degree of independence in changing the states of different variables. Recent experimental studies (Lloyd, Reid and Fealock, 1960; Lloyd, 1961; Reid, Lloyd, Brackett and Hawkins, 1961; Yntema and Muesser, 1960a, b, 1962) have established many relationships between the structure of the task and the accuracy of the subject's responses. I have tried to envisage a process which could reproduce the data from keeping track studies. This will be referred to as the occupancy model. Its assumptions may also be grouped into coding, storage, and retrieval assumptions:

#### Coding Rules

- (a) A message in a keeping track situation (e.g. (Animal of A) = (Dog) ) is represented internally by a state chunk which names the state and a variable chunk which names the variable.
- (b) Every chunk is marked with a serial number identifying the message which created it.

The coding rules imply a much more elaborate structure than that considered by Shepard. Stimuli are represented internally by elements. The elements, however, are differentiated. The type of information which they can transmit has been stated. The coding rules are used to establish boundary conditions; the real interest of the theoretician is in the storage system.

#### Storage Rules

(a) The subject has available m bins for storing chunks. Each bin can hold one and only one chunk.

(b) When a message is presented, each of its chunks is stored in a separate, randomly chosen bin.

(c) When a chunk is stored in a bin, that bin is given a reference number which states the bin into which the other chunk from the same message was stored.

(d) When a chunk is stored in a bin, the previous contents of the bin, if any, are lost.

(e) When a question is answered, the answer is stored as if it had been a message.

These storage rules set up a much more elaborate network of connections between stimulus and response than in the previous model. How complex such networks can be is illustrated in Table 2, which shows a possible state of a 10 bin memory after six messages about the animal, vegetable, and mineral of objects A and B have been received. Suppose the question "What is the animal of A?" were to be asked. This could easily be answered, bin one contains a message chunk naming the variable "Animal of A" and referring to bin eight. Bin eight, in turn, contains the state chunk naming the animal "dog". This chunk has the same serial number as the chunk in bin one, so the animal of A must be dog.

The question "What is the animal of B?" could be answered in exactly the same manner. The answer is "Cat" (bin ten). But suppose that a new message were to be received, changing memory

to the state shown in Table 3 . The chunk naming the variable "Animal of B" is no longer present. The correct answer, however, is still in bin ten. It is now a free state, a state chunk not connected to an appropriate variable chunk. Under some circumstances free states, since they represent recently received messages about some variable of a particular type, are useful in constructing guesses as to the correct answer. For instance, if there were only one object, then "cat" could be connected to only one variable, the animal of that object. In the example there is one chance out of two that "cat" was originally received as part of a message about the animal of A .

State chunks may also be wiped out by subsequent messages. Referring back to Table 2 we can see that message one had its variable chunk (vegetable of A) stored in bin six and its state chunk in bin nine. The state chunk was later superseded by the state chunk "lion" from message three. But "lion" should not be accepted as the name of the vegetable of A . Bin six might as well be empty, it transmits no useful information. In some situations in which it pointed to an allowable but wrong state chunk, say the chunk "beet" which actually was the vegetable of B , the contents of bin six could be confusing.

The actual answers found will depend on how the storage system is used. This is established by the retrieval assumptions.

#### Retrieval Rules

- (a) A message is coded as a single variable chunk.
- (b) The following searches are used to assemble an answer:

(b.1) Find a bin containing a chunk naming the variable in the question and pointing to a bin containing the state chunk from the same message. If more than two such pairs can be found, select the most recent pair. The state chunk names the answer.

(b.2) Find a state chunk which is in storage, is a plausible answer, but is not connected to any other plausible variable. If there is more than one such state, select the most recent one. The state chunk selected names the answer.<sup>4</sup>

(b.3) If neither steps b.1 or b.2 can be completed, choose an answer at random from the set of allowable answers.

These rules establish a well defined information storage and retrieval system. It is intuitively obvious that for a particular keeping track situation, (i.e. specific number of variables, objects, attributes, states, rate of change), the model implies a probability distribution for frequency of correct responses. I have been unable to find a mathematical statement of this distribution. Janet Kreuter, working under my direction, was able to construct the model physically by programming a digital computer. A Monte Carlo estimation of the distribution of responses was obtained by repeating 44 different keeping track experiments with simulated subjects.<sup>5</sup>

The use of a Monte Carlo technique proved crucial in evaluating the model. It demonstrated that while the model was correct in its broader aspects, it made certain consistent errors in prediction. The correlation between predicted and obtained results, computed over all the conditions investigated by Yntema and Musser,

is approximately .80 . (It varies somewhat, depending on how one defines an experimental condition and upon whether or not different sizes of memory are used to fit data obtained from different subjects.) The direction of influence of all independent variables on the dependent variable, probability of answering a question correctly, is predicted by the model. The magnitude of the effect of varying the different independent variables is rather poorly predicted.

Three major conclusions could be reached. The model did acceptably well in predicting the effect of extreme changes; from one to eight variables or from one object with six attributes to six objects with one attribute. It did not predict intermediate situations nearly so well. Some examples are shown in Table 4 . The model was spectacularly poor in predicting the effect of variation in the rate at which messages change the state of their variables. If two successive messages about the same variable assign it the same state, with high probability, people find the task quite easy. In fact, if the probability of a message changing the state of the variable is .25 human subjects will improve their probability of correct response by about .30 over otherwise comparable conditions in which the probability of change of state is 1 . The occupancy model predicts an improvement of about .10, clearly too small an increase. Finally, the model's performance rapidly decreases if several messages or questions have intervened between a question and the last message containing the information needed to answer it. The data from human subjects indicates a

similar deterioration, but not nearly as rapid a one.

This brief survey of results is sufficient to show that the complexities of the keeping track situation have by no means been explained by the occupancy model. They also demonstrate the advantage of having a model which makes precise statements. A start has been made on the job of explaining the keeping track data, it is quite clear what remains to be done. It may be possible to modify the occupancy model to predict the obtained data and suggest further experiments. On the other hand, it may be necessary to investigate a new class of models. This seems unlikely based on the information at hand.

Developing a model as a program also has the advantage of forcing one to be very specific about coding assumptions. The occupancy model is only applicable in situations in which the subject codes a message into exactly two chunks. The experimental procedure of Yntema and Mueser's studies makes the coding assumptions plausible, in some keeping track studies they are less plausible. We repeated some of the conditions of Yntema and Mueser's 1960b experiment, with the single difference that the task was a self paced one in which messages and questions were presented on a simple teaching machine. Subjects were interrogated after the experiment, they reported using a wide variety of memory "crutches," such as the construction of stories or visual images. One co-ed, trying to keep track of the jewel of A, B, and C, developed a subjective fashion parade in which she and her friends wore jewelry corresponding to currently active states. Other subjects,

however, adopted quite a different strategy. They simply repeated messages over and over to themselves. It seems reasonable to assume that the chunks created by such different techniques are themselves different. The same storage and retrieval model might work for each of our two groups of subjects, the same coding model is hardly appropriate.

#### Implications of the Model Building Approach

Objections to the model building approach can be addressed to different levels of interpretation. At the broadest level, a frequently heard objection is that it removes psychology from the study of memory. In creating artificial models, the psychologist may not be sufficiently concerned with the plausibility or generality of his theories. There seems to be no commitment to any of the real psychological issues; the mathematical psychologist is accused of having his nose to the ground, his eye on (his own) data, and no care for broader issues. This particular mathematical model maker is a straw man. In both mathematical and simulation model construction, there is a commitment to a theory of psychological processes. It has frequently been pointed out that stimulus sampling models are a way of specializing and describing Guthrie's conceptualization of learning. Shepard's model is an extension of a very similar model he proposed to account for the gradient of generalization (Shepard, 1958). There is perhaps an even greater commitment to process on the part of builders of simulation models. A computer program does not just spring out of one's head as a convenient way of summarizing facts. Nor need

it be restricted to special situations. Very general information processing models can be envisaged, some of these must be programmed before their implications can be realized. Two good examples of such general models are Feigenbaum's (1961) theory of general discrimination processes and Rosenblatt's (1962) plausible model of neurological reorganization in learning. Like the generalized stimulus sampling model, these theories are broader than the specific situation models I have dealt with here. They can be validated in two ways. It can be shown that the general principles behind them are sufficient to generate broad classes of phenomena of interest (e.g. retroactive interference, the serial position effect, all or none learning). They may also be tested in specific situations. When this is done, a micro-model, in the general spirit of the original theory but specialized to the particular experimental setting, must be developed. The examples I have discussed represent specializations.

The commitment to process is nowhere more clearly indicated than in those places in which a model fails to predict. Models may be inaccurate because they are generally bad, because they are generally good but contain a few steps which are not adequate representations of the corresponding steps in human information processing, or because the boundary conditions for testing the model are not satisfied. In the first case, failures to predict will be distributed randomly over the space of situations in which predictions were made. In the second case, failures will be clustered. Experiments which cannot be predicted by the model will be consistently related to some feature of the overall experi-

mental situation. This was the case when the occupancy model was tested in the keeping track situation. This sort of failure is interesting; it serves as a guide for future model building. At this point, the heuristic value of having a psychological process in mind while constructing equations or programs is most evident. The theorist must ask himself what sort of alternate sub-processes must be introduced which will respond differently than the present model to changes in variable x but will respond in the same way to changes in variables y and z, whose relations to the data have been explained.

There is always a danger that arguments over failure to satisfy boundary conditions will provide an amorphous catch-all for bad models. "It wasn't that the model is bad, just that the test wasn't appropriate." Such failures, however, may be informative. They indicate what boundary conditions must be satisfied. This, in turn, indicates what sort of models of peripheral processes are needed before adequate tests of the heart of the model can be made. The assumptions a psychologist makes about how the experiment looks to the subject are often quite inadequate to predict how the subject will structure a particular experimental task. We do not have good models of how human beings create chunks of information. This is particularly important when we study complex situations, such as the keeping track task.

The use of precise (analytic or Monte Carlo) descriptions of expected response distributions also has implications for

experimental tactics. The ancient paradigm of experimental vs. control loses most of its meaning. Instead of asking if treatment  $X$  is more or less effective than treatment  $Y$ , we are asking, "Does treatment  $X$  result in response distribution  $f(x)$ ?" Control treatment  $Y$  never enters into the picture. Conventional statistics are also not always appropriate. Every model is absolutely certain to be rejected in the sense that statistically significant deviations from it can be obtained. A more relevant question is "Which of several proposed models provides the best fit to the data?" Bayesian statistics may be used to guide us in our choice of that model which is the best prospect for further development. In constructing the occupancy model, a Bayesian technique was developed (Hunt, 1961) to do exactly this. Such choices are particularly appropriate in simulation studies where the cost of computing predicted data may be a significant part of the budget.

In making a non-statistical evaluation of a model, we give it positive credit if it is accurate and negative credit if it is complex. The utility of the model will be determined by a trade off between accuracy and complexity. The most common way of determining complexity is to ask how many of the model's parameters must be fitted from the data. As simulation models become more widespread, this practice will have to be re-examined. It is intuitively obvious that the occupancy model is a very complex one. Only one parameter, the number of storage bins, is determined by fitting to obtained data. By way of contrast,

Shepard's model contains five free parameters. Mathematical and simulation models must be described both by their free parameters and by the amount of computation they require. Precise definitions of "amount of computation" are needed.

In summary, memory must involve three stages: coding, storage, and retrieval. To have a model of memory means that one can state what each of these sub-processes are. The statement should be precise enough to be translated either into workable mathematical expressions or into a computer simulation program. The latter technique is especially suited for the study of complex memory phenomena. The former is better suited for extremely precise studies of simple situations.

## FOOTNOTES

1. I wish to express my thanks to Drs. Douwe Yntema and James MacQueen and to Miss Janet Kreuter, for discussion and comment on the ideas embodied in this paper. The responsibility for the presentation is, of course, my own.
2. Supported by the Office of Naval Research under task number NB-047-041 and the Western Management Science Institute under its grant from the Ford Foundation.
3. Dr. Shepard has not been involved in this re-interpretation, for which I bear sole responsibility. The particular examples selected have been chosen to illustrate a point rather than to imply that they are good or bad examples of this sort of theorizing.
4. A state chunk is considered "free" if its bin points to a bin containing another state chunk, a variable chunk naming the variable in the question, or a variable chunk which could not possibly be connected with the state chunk pointing to it (e.g. color pointing to animal). If a free state is found which is more recent than the state chunk selected in step b.1 the free state is selected as the answer with probability  $1/(\text{number of objects})$
5. The program was written in FORTRAN for the IBM 7090 computer.

Computations were performed at the Lincoln Laboratory, Massachusetts Institute of Technology, and at the Western Data Processing Center, University of California at Los Angeles.

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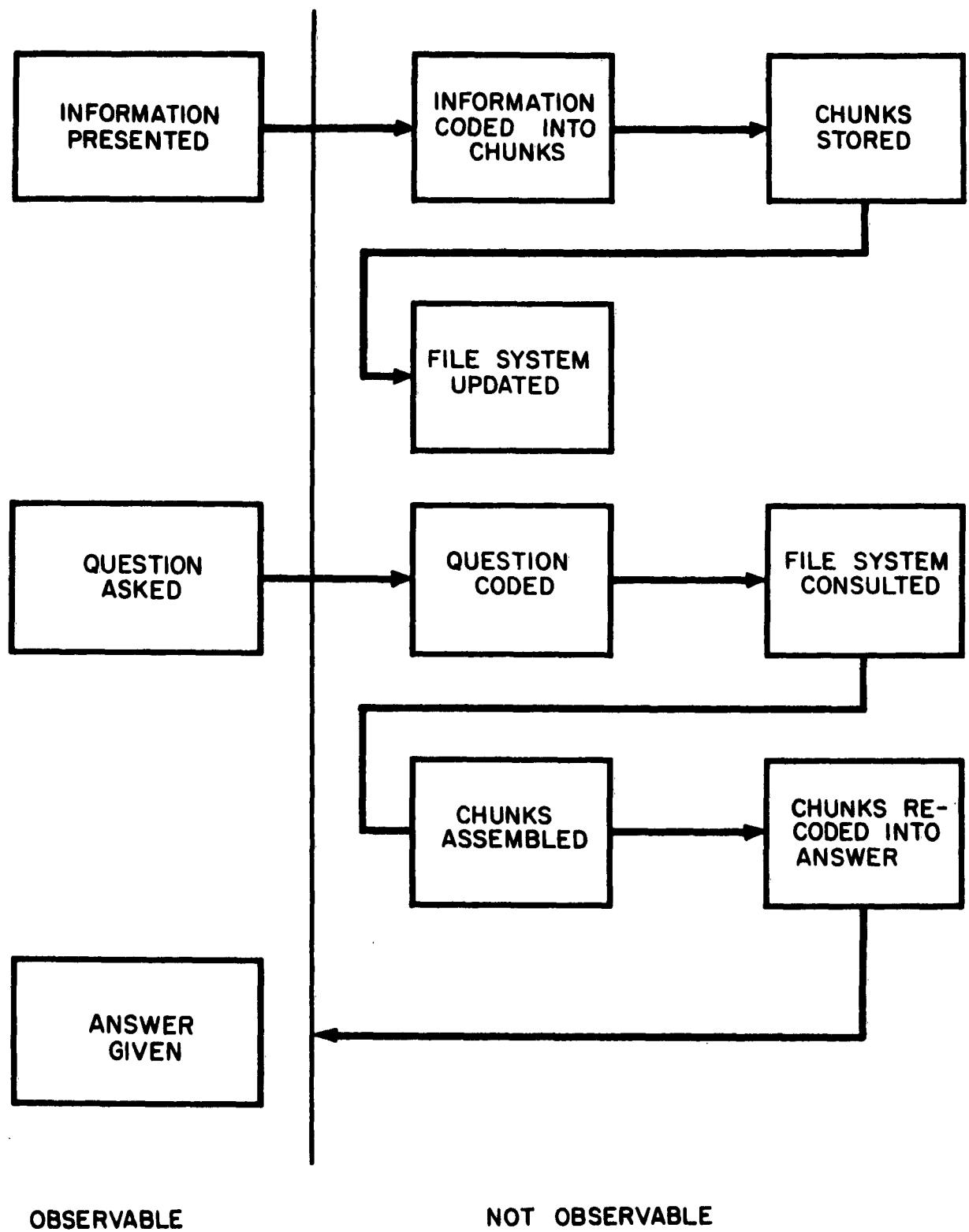
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FIGURE 1



STEPS IN INFORMATION STORAGE AND RETRIEVAL

TABLE 1

Probability of classifying a stimulus in the recognition array as a member of the response sequence

$$P_d(T) = \left\{ 1 - \left( \frac{1 - w(uV)^d - (1-w)(uV_c)^d}{1 + \frac{1 - u^T}{N(1-u)} - \frac{w(1 - (uV)^T)}{N(1-uV)} - \frac{(1-w)(1-(uV_c)^T)}{N(1-uV_c)}} \right)^B \right\}$$

The probability of classifying a new stimulus as old is determined by taking the limit as  $d \rightarrow \infty$

## DEFINITIONS

$N$  = number of stimuli under consideration in the experiment

$v$  = probability that a stimulus chunk migrates to a location representing a dissimilar stimulus on a given trial

$V$  =  $1-Nv$

$v_c$  = probability that a chunk migrates to a location representing a similar stimulus on a given trial,  $v_c > v$

$N_c$  = number of stimulus representations per cluster

$V_c = V - N_c(v_c - v)$

$u$  = probability that a chunk remains active on a given trial

$w$  = weight parameter

$B$  = response bias parameter

$d$  = number of presentations since the stimulus to be recognized was shown

$T$  = total number of stimuli presented

Derived expression for probability of identification of item in recognition array as old item using Shepard's model.

TABLE 2

BIN NUMBER	SERIAL NUMBER	CHUNK	REFERENCE BIN
1	5	Animal of A	8
2	6	Animal of B	10
3		(empty)	
4	4	Mineral of B	1
5		(empty)	
6	1	Vegetable of A	9
7	2	Diamond	10
8	5	Dog	1
9	3	Lion	4
10	6	Cat	2

Possible state of 10 bin memory after 6 messages about  
 Animal, Vegetable, Mineral of A and B

TABLE 3

BIN NUMBER	SERIAL NUMBER	CHUNK	REFERENCE BIN
1	5	Animal of A	8
2	7	Ruby	9
3		(empty)	
4	4	Mineral of B	1
5		(empty)	
6	1	Vegetable of A	9
7	2	Diamond	10
8	5	Dog	1
9	7	Mineral of A	2
10	6	Cat	2

Possible state of Example after message "Mineral of A = Ruby" has been received

TABLE 4

SITUATION			Probability that message changes state of variable	Obtained fraction right	Predicted fraction right
No. of objects	No. of attributes	No. of states			
VARIABLES CHANGED					
1	2	4	.75	.95	.91
1	3	4	.75	.88	.87
1	4	4	.75	.78	.72
1	6	4	.75	.69	.77
1	8	4	.75	.60	.64
2	1	4	.75	.64	.88
3	1	4	.75	.60	.72
4	1	4	.75	.47	.44
6	1	4	.75	.44	.38
8	1	4	.75	.35	.26
CONSTANT NUMBER OF VARIABLES					
1	6	8	.875	.75	.76
2	3	8	.875	.61	.51
3	2	8	.875	.56	.47
6	1	8	.875	.46	.48

Comparison of predicted and obtained performance in selected keeping track situations. All figures have been corrected for guessing.